NONLINEAR PROPERTIES OF THE TEMPERATURE GRADIENT IN PROBLEMS OF WAVE HEAT TRANSFER WITH MOVING BOUNDARIES

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High-intensity thermal phenomena for one-dimensional and self-similar two-dimensional versions are the object of investigation. The effect of the velocity of motion of the boundaries and the degree of nonstationarity of the thermal process on the behavior of the temperature gradient is considered. The conditions for the appearance of a gradient catastrophe on the boundary of a moving two-dimensional region are analyzed.

Problems of nonstationary heat transfer that arises in the studied body under the action of nonstationary elements of different nature (punching, surface hardening, welding, etc.) are of importance in technological thermophysics. The main results for this class of phenomena are obtained on the basis of the parabolic equation of heat conduction with a moving point energy source [1]. Within the framework of the linear hyperbolic problem, propagation of thermal waves from the moving edges of cracks in a solid body was studied in [2]. It is evident that for high-intensity thermal processes initiated in a body by some moving structural element we should study the effect of the following factors: 1) relaxation of the heat flux; 2) nonlinear thermophysical properties of the material; 3) nonlinear heat transfer on the moving boundary; 4) the finite dimensions of the moving region. As applied to this class of problems, in what follows nonlinear properties of the temperature gradient on one- and two-dimensional boundaries moving in a medium possessing a finite time of heat-flux relaxation are studied:

$$\mathbf{q} + \gamma \frac{\partial \mathbf{q}}{\partial t} = - \lambda \operatorname{grad} T . \tag{1}$$

The nonlinear hyperbolic equation of heat conduction has the form

$$c \left(\frac{\partial T}{\partial t} + \gamma \frac{\partial^2 T}{\partial t^2} \right) = \operatorname{div} (\lambda \operatorname{grad} T) , \quad c = c (T) , \quad \gamma = \gamma (T) , \quad \lambda = \lambda (T) . \tag{2}$$

It was derived in [3, 4] for media of the type of (1) by means of variational principles of the phenomena of nonlinear relaxational heat transfer.

This study is aimed at: 1) the development of an analytical approach to the investigation of the local properties of the temperature gradient in the vicinity of moving one- and two-dimensional self-similar boundaries; 2) the study of the effect of the velocity of boundary motion and the degree of nonstationarity of the thermal process on the behavior of grad T; 3) an analysis of the role of two-dimensional geometric factors in the formation of the structure of the relaxing thermal field.

1. One-Dimensional Process. At a constant temperature T_0 there is a stationary medium for which the one-dimensional temperature field is determined by Eqs. (1) and (2) written in the variables x, t. In nondimensionalization of these equations we use, in what follows, scales of quantities that admit invariance of the dimensional and dimensionless forms of notation, e.g., $q_b = \lambda_b T_b / x_b$, $\lambda_b = c_b x_b^2 / t_b$, etc. We consider a thermal process in the region with moving boundaries

$$t = 0 , \quad T = T_0 , \quad T_t = T_t^0 \equiv \text{const} , \quad x \in (- \infty, \infty) , \quad x^1 (0) = x_m (0) = 0 ;$$

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$$t \geq 0\,, \quad x = x^1(t)\,, \quad T = T^1(t)\,;$$

$$x = x_m(t)\,, \quad q = f(T_m)\,. \tag{3}$$

The last condition means that the boundary heat flux is a nonlinear function of the temperature $T_m = T(x_m, t)$. Here we assume $T^1(0) = T_m(0) = T_0$, $f(T_0) = 0$. We represent the heat flux in the form

$$q(x, t) = f(T) + \kappa(x, t)\,, \tag{4}$$

where $\kappa = \kappa(x, t)$ is an auxiliary function that should satisfy the conditions $\kappa(x, 0) = 0$, $\kappa(x_m, t) = 0$ so that relation (4) at $x = x_m$ would turn into the nonlinear condition of heat transfer (3). We write the equation for the heat flux in a divergence form using the functions

$$(\Lambda + H)_x + \Gamma_t = 0\,, \quad \Lambda(T) = \lambda(T)\,, \quad \Gamma(T) = \gamma(T)\,f(T)\,,$$

$$H_x = f + \kappa + \gamma\kappa_t\,, \tag{5}$$

where $H = H(x, t)$.

We introduce a scalar potential $\xi = \xi(x, t)$: $\xi_x = \Gamma$, $\xi_t = -(\Lambda + H)$, and we replace the variables x, t by ξ, t:

$$x(\xi, t) = \int \Gamma^{-1}(\xi, 0)\, d\xi + \int_0^t (\Lambda + H)\,\Gamma^{-1}\, dt\,, \quad \Gamma \neq 0\,. \tag{6}$$

Thus, we have three equations: the equation of heat transfer (2), Eq. (5), and the condition $(\xi_x)_t = (\xi_t)_x$ of equality of the mixed derivatives of second order for the scalar potential. Having transformed these equations by means of (6), we obtain a system denoted by $\Omega(\xi, t)$; its description is omitted here. For the type of transformations mentioned $\lambda\Gamma^2 \neq c\gamma(\Lambda + H)^2$, i.e., the velocity of the thermal wave is not equal to the velocity of ξ-line motion. We take $\xi = 0$ as the boundary of the region where heat transfer takes place according to nonlinear law (3):

$$\xi(x_m, t) = 0\,, \quad dx_m/dt = [(\Lambda + H)/\Gamma]_{\xi=0}\,.$$

The dependences $x^1 = x(\xi^1(t), t)$, $T^1 = T(\xi^1(t), t)$ correspond to the boundary $x = x^1(t)$ in the new variables. Having at our disposal the equations of $\Omega(\xi, t)$ and the consequences from them obtained by successive differentiation with respect to ξ, we find, using the Taylor formula, representations of the sought functions in the form of polynomials, e.g.,

$$T(\xi, t) = T_m(t) + \sum_{i=1}^{j} T_i\,(\xi^i/i!) + R_j\,, \quad \xi \in [0, \xi^1(t)] \subseteq [0, \xi_1)\,,$$

where R_j are the additional terms in the Lagrange form, $0 < \xi_1 < 1$. We note that $H_m(t)$ is an arbitrary function and $\kappa_m = 0$. Convergence of the Taylor series can be shown for processes of highly intense heat transfer, when the second term in the left-hand side of Eq. (2) is predominant, i.e., the wave mechanism of heat transfer prevails over diffusion. We assume that: 1) the functions $c(T)$, $\lambda(T)$, $\gamma(T)$, $f(T)$ are analytic; 2) at the initial instant of time $T_0 = 0$, $\gamma(T_0) = 0$, and for $t > 0$ we have $\gamma(T) > 0$; 3) the functions $T_m(t)$, $T_1(t) \equiv (T_\xi)_m$, $H_m(t)$ are analytic and are represented in the form of power series with a nonzero radius of convergence, and $T_m(0) = 0$, $T_1(0) = 0$. Then the coefficients $T_i(t)$, $H_i(t)$ are analytic functions possessing the properties $T_i(0) = 0$, $H_i(0) = 0$, $|T_i(t)| \leq L_1 < \infty$, $|H_i(t)| \leq L_2 < \infty$, $i \geq 1$, $0 \leq t \leq t_0 < \infty$. Then, the corresponding Taylor series are analytic functions. To determine the heat flux by formula (4), we must solve Eq. (5). It is not difficult to see that when the asymptotic equalities

$$t \to 0\,, \quad f(T)/\gamma(T) \to 0\,, \quad \xi^1(t)/\gamma_m(t) \to 0\,, \quad T_1(t)/\gamma_m(t) \to 0\,,$$

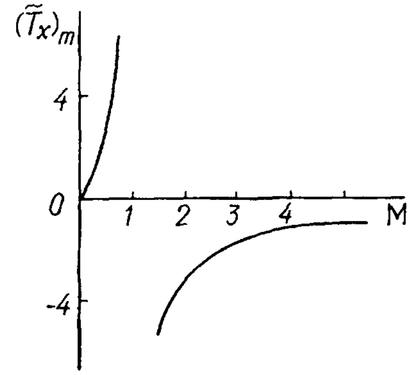


Fig. 1. Dependence of the extremum value of the temperature gradient on the Mach number.

are fulfilled, the Taylor series $\kappa = \sum_{i=1}^{\infty} \kappa_i \xi^i / i!$, which gives the solution of Eq. (5), converges.

We illustrate the results obtained. Let $\lambda = \lambda_0 T$, $\gamma = \gamma_0 T$, $c \equiv$ const, $f = f_0 T^a$, and the line $\xi_m = 0$ propagate with a constant velocity $dx_m/dt = V > 0$. We consider thermal processes on subsonic, $\mathrm{M} \equiv V/w < 1$, and supersonic, $\mathrm{M} > 1$, boundaries. Here gasdynamic terminology and a thermal analog of the Mach number are used. The functions $T_m(t)$, $T_1(t)$, $T_2(t)$ are related by one hyperbolic equation of heat conduction, so that two of them are arbitrary. We assume that $T_1 = a_1 t^{n-1}$ and, to satisfy the conditions of convergence, we represent T_2 in the form of the sum $T_2 = B + \theta_2$, where $B(t)$ corresponds to the terms in the equation of heat conduction involving the derivative Vd/dt, and $\theta_2(t)$ to the remaining terms. Then, we have

$$\theta_2 = \left[\lambda \Gamma^2 (1 - \mathrm{M}^2)\right]_m^{-1} \left[c\gamma \frac{d^2 T_m}{dt^2} + \Gamma T_1^2 (\dot{\Gamma} \lambda (\mathrm{M}^2 - 1) - \lambda_0 \Gamma)\right]_m ,$$

$$B = a_1^2 (b_0 t^{n-2} + b_1 t^{k+n-2} + b_2 t^{k+n-1} + \dots), \quad k > 0 .$$

A specific assignment of the function $B(t)$ uniquely affects the temperature $T^1(t)$ of the opposite boundary $\xi = \xi^1(t)$. Next, for the sake of brevity of the formulas, in the last expression we restrict ourselves to the first two terms: $b_2 = 0$, $b_3 = 0$, etc.; b_0, b_1 are arbitrary nonzero numbers. Thus, for the temperature and the temperature gradient on the boundary $x_m = Vt$ we find the formulas

$$T_m^{a+1} = 2(a+1) V B_1 t^k \left[a \gamma_0 f_0 (V^2 - w^2) a_1 B_0 (B_0 + B_1 t^k)\right]^{-1} ,$$

$$(T_x)_m = 2 V t^{n-1} \left[(w^2 - V^2)(B_0 + B_1 t^k)\right], \tag{7}$$

$$B_0 = b_0 (1 - n) > 0 , \quad B_1 = b_1 (1 - n + k) > 0 , \quad n > 3 , \quad k > 4 , \quad a > 1 ,$$

$$f_0 > 0 ,$$

$$(\mathrm{M}^2 - 1) a_1 > 0 , \quad \Gamma_m < 0 .$$

Consequently: 1) the temperature gradient is opposite to the vector of the velocity of a supersonic boundary, and in a subsonic process these directions are the same; 2) the modulus of the temperature gradient is a nonmonotonic function of time; 3) expression (7) does not depend on the law of heat transfer (3); it is characterized by the values of the velocities V, w and the parameters B_0, B_1 regulating the temperature $T^1(t)$ of the opposite boundary; the parameters f_0, a of nonlinear heat transfer affect the temperature $T_m(t)$ considerably. For the class of one-dimensional thermal processes presented here no gradient catastrophe [5] appears.

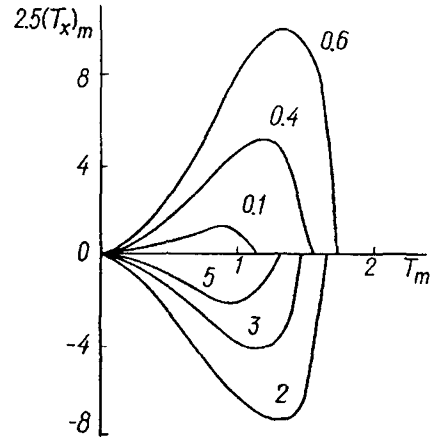


Fig. 2. Temperature dependence of the temperature gradient on the boundary $x_m = Vt$; the figures at the curves are the values of M.

We present an example. We take: $c = 1$; $\lambda_0 = 1$; $f_0 = 1$; $\gamma_0 = 1$; $a = 4$; $a_1 = \pm 0.4$; $k = 11$; $n = 4$; $B_0 = 1/3$; $B_1 = 1/8$. Calculations showed that the extremum value of the temperature gradient is a monotonically increasing function of the Mach number in both sub- and supersonic processes (see Fig. 1). The fact that with a monotonic dependence of the boundary heat flux on the temperature the temperature gradient changes nonmonotonically (Fig. 2) is important. In a subsonic case when $V > 0$ this dependence has a maximum, and in a supersonic case a minimum.

2. Two-Dimensional Region of Finite Dimensions. For the thermophysical parameters we take a power-law dependence on the temperature

$$c = c_0 T^s , \quad \lambda = \lambda_0 T^n , \quad \gamma = \gamma_0 T^{\varepsilon} , \quad \varepsilon = s - n ,$$

and we consider a plane self-similar version

$$\alpha = x + l_1 \tau , \quad \beta = y + l_2 \tau , \quad \tau = \ln (t + t_0) , \quad 0 \le t \le t_1 < \infty , \quad t_0 > 0 ,$$

$$T = (t + t_0)^k \, \Theta (\alpha, \beta) , \quad q_1 = (t + t_0)^{kn+k} \, u (\alpha, \beta) , \quad \varepsilon k = 1 ,$$

$$q_2 = (t + t_0)^{kn+k} \, v (\alpha, \beta) , \quad w_0^2 = \lambda_0 / (c_0 \gamma_0) .$$

According to (2), the type of equation for $\Theta(\alpha, \beta)$ depends on the sign of the quantity $l_1^2 + l_2^2 - w_0^2 \Theta^{-2\varepsilon}$. We have ellipticity for a negative value and hyperbolicity for a positive value. The solution is constructed in a closed plane region bounded by the lines $\beta = \beta^1(\alpha)$, $\beta = \beta_m(\alpha)$. On one of the boundaries the condition of nonlinear heat transfer is taken:

$$q_{1m} = \sum_{i=1}^{L_1} \eta_i T_m^{a_i} (t + t_0)^{k(n+1-a_i)} , \quad q_{2m} = \sum_{i=1}^{L_2} \zeta_i T_m^{b_i} (t + t_0)^{k(n+1-b_i)} . \tag{8}$$

The procedure of construction of the solution in the vicinity of this boundary is basically similar to the one-dimensional case and is not detailed here. We note only that for the components of the vector of the heat flux the relations

$$u = u_m (\Theta) + g (\alpha, \beta) , \quad v = v_m (\Theta) + h (\alpha, \beta) ,$$

based on boundary conditions (8), are introduced, and the auxiliary functions should satisfy the conditions $g(\alpha, \beta_m) = 0$, $h(\alpha, \beta_m) = 0$. Then one of the equations for a component of the heat flux is written in divergence form, and a potential function $\xi = \xi(\alpha, \beta)$ is introduced. Next, the polar coordinates r, φ are used: $\xi = r \cos \varphi$, $\alpha = r \sin$

φ. As a result we obtain a system of four equations for the functions Θ, H, g, h of the arguments r, φ; we call it the system $\Omega(r, \varphi)$. The role of the auxiliary function H is the same here as in the one-dimensional case (5). The value $r_m = 1$ is taken as the boundary of the region where nonlinear heat transfer (8) occurs, and we write it in the parametric form $\beta = \tilde{\beta}_m(\varphi)$, $\alpha = \sin\varphi$. The equations of $\Omega(r, \varphi)$ and their differential corollaries obtained by differentiation with respect to r allow one to construct the solution in the vicinity of $r = 1$ in the form of polynomials using the Taylor formula. The temperature mode on the second closed boundary $\beta = \beta^1(\alpha)$, $\Theta = \Theta^1(\alpha)$ is uniquely determined by the choice of the arbitrary functions involved in the solution.

We consider the case $\Theta_m \equiv \text{const}$ in more detail:

$$\tilde{\beta}_m(\varphi) = R_m^{-1}\cos\varphi + \mu(\sin\varphi - 1) + B(\varphi)\,, \quad \alpha_m(\varphi) = \sin\varphi\,, \tag{9}$$

$$B(\varphi) = R_m^{-1}\int_{\pi/2}^{\varphi} H_m(\varphi)\cos\varphi\, d\varphi\,, \quad \mu = U_m/R_m \equiv \text{const}\,,$$

$$U = \lambda_1 m_0 \Theta^{n-\varepsilon+1} + \gamma_0 l_2(u_m + m_0 v_m)\,, \quad m_0 \neq 0\,,$$

$$R = \lambda_1\Theta^{n-\varepsilon+1} + \gamma_0 l_1(u_m + m_0 v_m)\,, \quad \lambda_1 = \lambda_0/(n - \varepsilon + 1)\,.$$

The arbitrary function $H_m(\varphi)$ is selected so that the line (9) has two smooth branches that converge at $\alpha = \pm 1$ and bound a closed region of the plane (α, β):

$$B\left(\frac{\pi}{2}\right) = -2\mu\,, \quad B\left(\frac{3\pi}{2}\right) = 2\mu\,, \quad B(\pi) = -B(2\pi) = b_1^2 + \mu + R_m^{-1}\,,$$

$$B\left(\frac{5\pi}{2}\right) = -2\mu\,, \quad 2\mu + b_1^2 > 0\,, \quad b_1 \equiv \text{const}\,.$$

For example, we can take

$$-B(\varphi) = 2\mu(\sin\varphi)^{1+2n_1} + (b_1^2 + \mu + R_m^{-1})(\cos\varphi)^{1+2n_2}\,, \quad n_1 \geq 0\,, \quad n_2 \geq 1\,, \tag{10}$$

where n_1, n_2 are integers. The boundary line (9) has the form

$$\alpha^2 + [\beta - \alpha\mu + \mu - B(\varphi)]^2 R_m^2 = 1\,.$$

For the second boundary $\beta = \beta^1(\alpha)$ we take $r^1 = 1 + \delta(\varphi)$, $|\delta(\varphi)| \leq \delta_1 < 1$, $\delta(\varphi_i) = 0$, $\dot{\delta}(\varphi_i) = 0$, $\varphi_i = i\pi/2$, $i = 1, \ldots, 5$. Then we have that the lines of both boundaries "stick together" at $\varphi = \varphi_i$ and have a common tangential line at these points. The points of the plane (α, β)

$$B_2(1, -2\mu)\,; \quad A_1(0, b_1^2)\,; \quad B_1(-1, 0)\,; \quad A_2(0, -b_1^2 - 2\mu)\,; \quad b_1^2 + 2\mu > 0\,. \tag{11}$$

correspond to the values $\varphi = \varphi_i$. For the analytical functions $\Theta_m(\varphi)$, $\Theta_1(\varphi)$, $\delta(\varphi)$ the conditions of boundedness and the properties

$$|\Theta_m(\varphi)| \leq N_1 < \infty\,, \quad |\Theta_1(\varphi)| \leq N_2 < \infty\,, \quad |\delta(\varphi)| \leq \delta_1 < 1\,, \quad \varphi \in \left[\frac{\pi}{2}\,, \frac{5\pi}{2}\right]\,;$$

$$\left|\lim_{\varphi\to\varphi_j}(\dot{\Theta}_1/\cos\varphi)\right| < \infty\,, \quad \lim_{\varphi\to\varphi_j}(\delta/\cos\varphi) = 0\,, \quad \varphi_j = \frac{\pi}{2}(2j + 1)\,, \quad j = 0, 1, 2\,.$$

should be fulfilled. Then we have

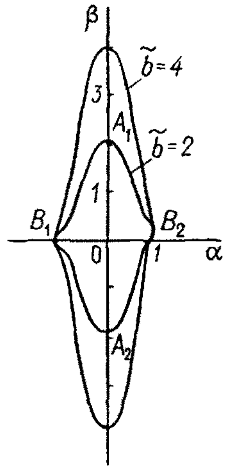


Fig. 3. Plane two-dimensional region of elliptical shape.

$$\lim_{\varphi \to \varphi_j} H(r, \varphi) = H_m(\varphi_j), \quad \left| \lim_{\varphi \to \varphi_j} \Theta_\varphi H_r \right| < \infty .$$

Thus, we have that for the functions $\Theta(r, \varphi)$, $g(r, \varphi)$, $h(r, \varphi)$ the Taylor series of the form

$$\sum_{k=1}^{\infty} \Theta_k(\varphi)\,(r-1)^k/k!, \quad \varphi \in \left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$$

converge for $|r-1| \le \delta_1 < 1$.

We analyze the version $s = 0$, $n = 1$, $a_1 = 2$, $b_1 = 2$, for which $\lambda = \lambda_0 T$, $\gamma = \gamma_0/T$, $c \equiv$ const.

The boundary of the region cools off: $T_m = \Theta_m/(t + t_0)$, $t \in [0, t_1]$, and heat transfer occurs according to the law $q_{1m} = \eta T_m^2$, $q_{2m} = \zeta T_m^2$.

As an example we consider the properties of a thermal field on a boundary (9), (10) of elliptical form that is obtained for $R_m \ge 1$, $|\mu| < 1$, $\tilde{b} = b_1^2 + \mu \ge 2$, where $\tilde{b}$, according to (11), is the relative elongation of the region, i.e., the ratio of its length A_1A_2 to its width, equal to 2 (Fig. 3). This boundary moves translationally along the OY axis with a velocity $V_m = -l_2/(t + t_0) > 0$, $l_1 = 0$. Applying the analog of the Mach number $\mathrm{M} = V_m/w(T_m)$, we distinguish subsonic ($\mathrm{M} < 1$), sonic ($\mathrm{M} = 1$), and supersonic ($\mathrm{M} > 1$) regimes of region motion. The equation of heat transfer (2) entering the system $\Omega(r, \varphi)$ relates the two functions $\Theta_1(\varphi)$, $\Theta_2(\varphi)$ on the boundary. Satisfying the conditions of convergence, we take $\Theta_2 = \tilde{\Theta}_2 + B(\alpha_m)\cos^2\varphi$. The function $B(\alpha) = B_0 + B_1\alpha + \ldots$ is arbitrary; the quantity $\tilde{\Theta}_2$ is expressed in terms of Θ_m, Θ_1, $\dot{\Theta}_1$ (the description is omitted). Then we have

$$p_1 = \sin\varphi - (U_m + H_m)\cos\varphi, \quad p_2 = \cos\varphi + (U_m + H_m)\sin\varphi,$$

$$H_m = 3(\tilde{b}R_m + 1)\cos\varphi\sin\varphi - 2U_m, \quad H_\varphi = 3(\tilde{b}R_m + 1)\cos 2\varphi,$$

$$\Theta_1 = \frac{B(\alpha_m)\cos^2\varphi\,[(\mathrm{M}^2 - 1)R_m^2\cos^2\varphi - p_1^2]}{\mathrm{M}V_m c w_0 \lambda_0^{-1}(1 - 3\gamma_m\Theta_m^{-1})\cos\varphi - (\mathrm{M}^2 - 1)V_m^2\sin^2\varphi - H_\varphi + p_2^2}.$$

The line $r = 1$ possesses the property that along it the derivative of the temperature in the direction tangential to the boundary equals zero. In this class of solutions the temperature gradient at $r = 1$ is directed along the normal to the boundary of the region and is characterized by the derivative

$$\left(\frac{\partial\Theta}{\partial n}\right)_m = \Theta_1(R_m\cos\varphi - p_1\dot{\beta}_m)\,[1 + (\dot{\beta}_m)^2]^{-\frac{1}{2}}, \quad \dot{\beta}_m = \frac{d\beta_m}{d\alpha}.$$

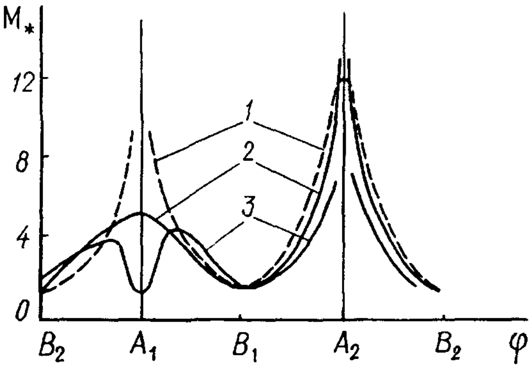
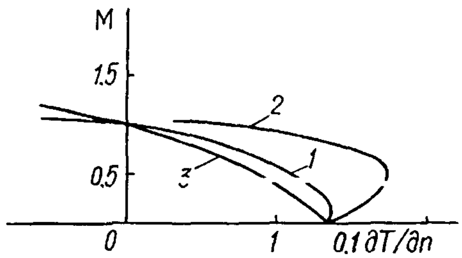


Fig. 4. Values of the critical Mach number on the region boundary: 1) γ^0 = 0.2, 2) 1.0, 3) 10.



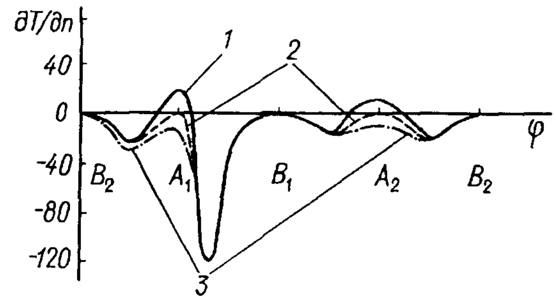


Fig. 5. Dependence between the Mach number and the temperature gradient in a "shockless" mode of region motion: front point A_1 [1) $\gamma^0 = 1$, 2) 10] and rear point A_2 [3) $\gamma^0 = 1$].

Fig. 6. Distribution of the temperature gradient along the boundary of a two-dimensional region at $\gamma^0 = 1$, $M_* = 1.30$ for three modes of motion: 1) M = 0.1, 2) 1, 3) 1.25.

We present results of numerical calculations at: $\Theta_m = 3$; $c = 1$; $\lambda_0 = 1$; $t_0 = 3$; $\mu = 0.1$; $\tilde{b} = 2$; $B_m = 1 + 0.2 \sin \varphi$. The initial (at $t = 0$) dimensionless time of relaxation $\gamma_m(0) = \gamma^0$ characterizes the degree of nonstationarity of the process; we have $\gamma^0 < 1$ if the time scale is larger than the relaxation time; $\gamma^0 > 1$ if the process is rapid.

We note the main properties of the boundary temperature gradient. For the prescribed shape of the boundary (the parameters μ, $\tilde{b}$) and a fixed γ^0, in motion of the region at a velocity V_m corresponding to the critical value of the Mach number M_*, a gradient catastrophe appears at one or several points of the boundary: $\partial T/\partial n$ becomes infinitely large. This means that the structure of the temperature field changes qualitatively: the field becomes discontinuous, and a shock thermal wave appears [5]. For nonlinear media possessing thermal relaxation the problem of the appearance, stability, and propagation of these shock waves was studied in [6-11]. At the front point A_1 we have

$$M_*^{(A_1)} = \lambda_0 \left[3(\tilde{b}R_m + 1) - 1\right] / \left[cw_0 R_m (3\gamma_m \Theta_m^{-1} - 1)\right]. \tag{12}$$

At the side points B_1, B_2 (Fig. 3) the attainment of the critical value does not depend on the time of relaxation

$$M_*^{(B_1)} = M_*^{(B_2)} = 1 + \left[U_m + 3(\tilde{b}R_m + 1)\right] R_m^{-2}. \tag{13}$$

We note that the right-hand sides in (12), (13) are linearly dependent on the relative elongation $\tilde{b}$. The distribution $M_*(\varphi)$ along the region boundary is shown in Fig. 4 for three modes that differ by the degree of process nonstationarity. These results show that at a fixed γ^0 a finite interval of Mach numbers $M \in [0, M_*^{\min})$ exists in which a gradient catastrophe does not appear: "shockless" motion of the region. In a slow process ($\gamma^0 = 0.2$) a gradient catastrophe does not appear at the front point, and the minimum value M_*^{min} corresponds to the extreme side points of the profile; for the rear point A_2 the quantity M_* is finite (line 1 in Fig. 4). If $\gamma^0 = 1$, then $M_*(\varphi)$ shifts to the point A_2 (line 2 in Fig. 4). For a rapid process ($\gamma^0 = 10$) M_*^{min} is attained at the front point A_1; at the points of the branches B_1A_2 and B_2A_2 a gradient catastrophe appears at any finite $M > M_*^{(B_1)} = M_*^{(B_2)}$; at the point A_2 the function $M_*(\varphi)$ has a discontinuity (line 3 in Fig. 4).

We also note that in the "shockless" range the dependence of $\partial T/\partial n$ on the Mach number is monotonic in a slow process, and at $\gamma^0 > 1$ this relation has a markedly nonmonotonic character (Fig. 5). The effect of the Mach number on the temperature gradient is most appreciable at the front and rear points of the boundary profile (Fig. 6). In passage from subsonic to supersonic "shockless" motion of the region, the direction of the temperature gradient at these points changes to the opposite; this phenomenon has already been noted in the one-dimensional case (see Fig. 2). At other points of the profile the effect of M on grad T is less substantial: the three graphs in Fig. 6 have pronounced quantitative differences near the points A_1 and A_2.

Let us sum up. High-intensity heat transfer in the vicinity of moving sub- and supersonic boundaries is accompanied by interesting physical phenomena. In the case of a plane self-similar closed boundary the relaxation properties of the temperature gradient manifest themselves in a different manner for rapid and slow processes. Formulas are obtained for the critical Mach number upon attainment of which a gradient catastrophe begins. It is shown that there is a fundamental difference between the front point and the extreme side points of the boundary: for the latter M_* does not depend on the time of thermal relaxation. The properties of the "shockless" mode of motion of one- and two-dimensional self-similar boundaries are studied.

NOTATION

T, temperature; q, vector of the specific heat flux; t, time; x, y, rectangular Cartesian coordinates; λ, coefficient of thermal conductivity; c, volumetric specific heat capacity; γ, time of heat-flux relaxation; w, velocity of heat propagation; M, thermal Mach number; α, β, self-similar variables; $T_2 = (\partial^2 T/\partial \xi^i)_m$; $H = (\partial^i H/\partial \xi^i)_m$; $\Theta_i = (\partial^i \Theta/\partial r^2)_m$; $g_i = (\partial^i g/\partial r^i)_m$; ξ, scalar potential; Θ, u, v, g, h, unknown functions of the self-similar variables α, β; a, a_i, b_i, n, k, s, l_1, l_2, η_i, ζ_i, η, ζ, γ_0, w_0, f_0, t_0, L_1, L_2, constant parameters characterizing the thermophysical properties of the medium and the conditions of heat transfer on the moving boundary of the region. Subscripts and superscripts: dot above the symbol of a function, ordinary differentiation with respect to its argument; independent variable as a subscript, partial differentiation; b, scale of the quantity; m, values of the function on the studied moving boundary; 1 as a superscript, values of the function on the second boundary.

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